Sample Question Paper - 30
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours *Maximum Marks : 40*

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Solve the following quadratic equation for x :

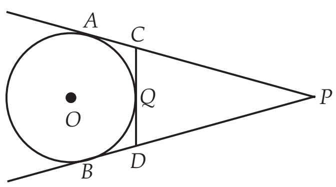
$4x^2 - 4a^2x + (a^4 - b^4) = 0$

2. How many two-digit numbers are divisible by 3?

3. If O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then find $\angle POQ$.

OR

In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.



4. Data of 'missed catches' for the 40 matches played by a player is as follows :

Number of missed catches in a match	0-3	3-6	6-9	9-12	12-15
Number of matches	15	16	3	4	2

Calculate the mean number of catches missed by him.

5. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base of the cone and that of hemisphere is 6 cm and the height of cone is 4 cm. Calculate the surface area of the toy. [Take $\pi = 3.14$]

OR

A toy is in the shape of a cone mounted on a hemisphere of same base radius. If the volume of the toy is 231 cm^3 and its diameter is 7 cm, then find the height of the toy. $\left[\text{Use } \pi = \frac{22}{7}\right]$

6. The mean of a set of numbers is $\bar{x}$. If each number is multiplied by k, then find the mean of the new set.

SECTION - B

7. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k - 1)x + x^2 = 0$ has equal roots. Hence, find the roots of the equation.

OR

Find two consecutive positive integers, the sum of whose squares is 61.

8. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.

9. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$)

10. Draw a circle of radius 2.4 cm. Take a point P on it. Without using the centre of the circle, draw a tangent to the circle at point P.

SECTION - C

11. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3}$ = 1.732]

12. The mode of the following data is 36. Find the missing frequency x in it.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	x	16	12	6	7

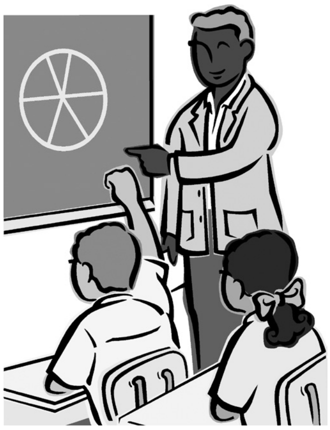
OR

If the median of the following frequency distribution is 32.5, then find the values of f_1 and f_2.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	**Total**
Frequency	f_1	5	9	12	f_2	3	2	40

Case Study - 1

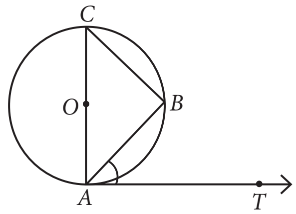
13. For class 10 students, a teacher planned a game for the revision of chapter circles with some questions written on the board, which are to be answered by the students. For each correct answer, a student will get a reward. Some of the questions are given below.



Answer these questions to check your knowledge.

(i) If PA and PB are two tangents drawn to a circle with centre O from P such that $\angle PBA = 50°$, then find the measure of $\angle OAB$.

(ii) In the adjoining figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 55°$, then find the measure of $\angle BAT$.



Case Study - 2

14. Isha's father brought an ice-cream brick, empty cones and scoop to pour the ice-cream into cones for all the family members. Dimensions of the ice-cream brick were $(30 \times 25 \times 10)$ cm^3 and radius of hemi-spherical scoop was 3.5 cm. Also, the radius and height of cone were 3.5 cm and 15 cm respectively.

Based on the above information, answer the following questions.

(i) Find the quantity of ice-cream in the brick (in litres).

(ii) Find the minimum number of scoops required to fill one cone upto brim.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. We have, $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$\Rightarrow (2x)^2 - 2(2x)a^2 + (a^2)^2 - (b^2)^2 = 0$

$\Rightarrow (2x - a^2)^2 - (b^2)^2 = 0$

$\Rightarrow (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$\Rightarrow 2x - a^2 + b^2 = 0$ or $2x - a^2 - b^2 = 0$

$\Rightarrow 2x = a^2 - b^2$ or $2x = a^2 + b^2$

$$\Rightarrow x = \frac{a^2 - b^2}{2} \text{ or } x = \frac{a^2 + b^2}{2}$$

2. Two-digit numbers which are divisible by 3 are 12, 15, 18, …, 99, which forms an A.P. with first term $(a) = 12$, common difference $(d) = 15 - 12 = 3$ and last term (l) or n^{th} term $(a_n) = 99$

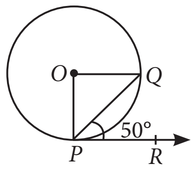
$\therefore a + (n - 1)d = 99$

$\Rightarrow 12 + (n - 1)3 = 99 \Rightarrow 3n = 99 - 9$

$$\Rightarrow n = \frac{90}{3} = 30$$

Thus, there are 30 two-digit numbers which are divisible by 3.

3. $\because PR$ is a tangent to the circle.



$\therefore OP \perp PR$

$\Rightarrow \angle OPR = 90°$

$\Rightarrow \angle OPQ + \angle QPR = 90°$

$\Rightarrow \angle OPQ = 90° - 50° = 40°$

Now, $OP = OQ$ (Radii of circle)

$\Rightarrow \angle OPQ = \angle OQP = 40°$

In ΔOPQ, $\angle OPQ + \angle OQP + \angle POQ = 180°$ [By angle sum property]

$\Rightarrow \angle POQ = 180° - 40° - 40° = 100°$

OR

Tangents drawn from an external point are equal in length.

$\therefore QC = CA$, $QD = BD$ and $PA = PB$

Since, $QC = QD = 3$ cm [Given]

$\Rightarrow CA = BD = 3$ cm

Also, $PC = PA - AC$

$\Rightarrow PC = (12 - 3)$ cm $= 9$ cm [Given, $PA = 12$ cm]

Similarly, $PD = 9$ cm

$\therefore PC + PD = 9 + 9 = 18$ cm

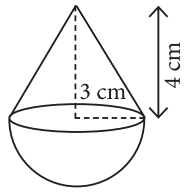
4. The frequency distribution table from the given data can be drawn as :

Missed catches	Class marks (x_i)	Frequency (f_i)	f_ix_i
0-3	1.5	15	22.5
3-6	4.5	16	72
6-9	7.5	3	22.5
9-12	10.5	4	42
12-15	13.5	2	27
		$\Sigma f_i = 40$	$\Sigma f_ix_i = 186$

$$\therefore \text{Mean} = \frac{\sum f_ix_i}{\sum f_i} = \frac{186}{40} = 4.65$$

5. Radius of the base of the cone and hemisphere (r)

$$= \frac{6}{2} = 3 \text{ cm}$$



Height of cone $(h) = 4$ cm

Slant height of cone (l)

$= \sqrt{r^2 + h^2} = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ cm

Total surface area of toy

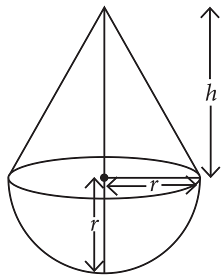
= Curved surface area of hemisphere + Curved surface area of cone

$= 2\pi r^2 + \pi rl = \pi r(2r + l) = 3.14 \times 3(2 \times 3 + 5)$

$= 3.14 \times 3 \times 11 = 103.62$ cm^2

OR

Let h be the height of the cone and r be its radius.



Given, $r = (7/2)$ cm $= 3.5$ cm

Volume of the toy $= 231$ cm^3

$\Rightarrow$ Volume of cone + Volume of hemisphere $= 231$ cm^3

$$\Rightarrow \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = 231$$

$$\Rightarrow \frac{1}{3}\pi r^2 (h + 2r) = 231$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(h + 2 \times \frac{7}{2}\right) = 231 \Rightarrow \frac{77}{6}(h + 7) = 231$$

$$\Rightarrow h + 7 = \frac{231 \times 6}{77}$$

$\Rightarrow h + 7 = 18 \Rightarrow h = 11$ cm

$\therefore$ Height of cone is 11 cm

Total height of toy = Height of cone + radius of hemisphere $= 11 + 3.5 = 14.5$ cm

6. Let the numbers are $x_1, x_2, \ldots, x_n$.

$$\therefore \text{Mean} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \bar{x} \quad \ldots(i)$$

When given numbers are multiplied by k, then new observations are $kx_1, kx_2, \ldots, kx_n$.

New mean $= \frac{kx_1 + kx_2 + ... + kx_n}{n}$

$= \frac{k(x_1 + x_2 + ... + x_n)}{n} = k\bar{x}$ (From (i))

7. We have, $kx^2 + 1 - 2(k - 1)x + x^2 = 0$

or $(k + 1)x^2 - 2(k - 1)x + 1 = 0$ ...(i)

Since, roots are equal. $\therefore$ $D = 0$

$\Rightarrow \{-2(k - 1)\}^2 - 4 \times (k + 1) \times 1 = 0$

$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 0$

$\Rightarrow 4k^2 - 12k = 0 \Rightarrow 4k(k - 3) = 0$

$\Rightarrow k = 0$ or $k - 3 = 0 \Rightarrow k = 3$

$\Rightarrow k = 3$ (Non-zero value of k)

Substituting the value of k in (i), we get

$(3 + 1)x^2 - 2(3 - 1)x + 1 = 0$

$\Rightarrow 4x^2 - 4x + 1 = 0 \Rightarrow 4x^2 - 2x - 2x + 1 = 0$

$\Rightarrow 2x(2x - 1) - 1(2x - 1) = 0 \Rightarrow (2x - 1)(2x - 1) = 0$

$\Rightarrow 2x - 1 = 0$ or $2x - 1 = 0 \Rightarrow x = \frac{1}{2}, \frac{1}{2}$

OR

Let the two consecutive positive integers be x and $x + 1$.

According to question, $x^2 + (x + 1)^2 = 61$

$\Rightarrow x^2 + x^2 + 2x + 1 = 61$

$\Rightarrow 2x^2 + 2x = 60 \Rightarrow x^2 + x - 30 = 0$

$\Rightarrow (x - 5)(x + 6) = 0$

$\Rightarrow x = 5$ or $x = -6$

$\Rightarrow x = 5$ [Since x is a positive integer]

And $x + 1 = 6$

$\therefore$ The two consecutive positive integers are 5 and 6.

8. Let a be the first term and d be the common difference of the A.P.

Sum of n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

We have, $S_7 = 49$

$\Rightarrow \frac{7}{2}[2a + 6d] = 49$

$\Rightarrow 14a + 42d = 98 \Rightarrow a + 3d = 7$ ...(i)

and $S_{17} = 289$

$\Rightarrow \frac{17}{2}[2a + 16d] = 289$

$\Rightarrow 34a + 272d = 578 \Rightarrow a + 8d = 17$ ...(ii)

On solving (i) and (ii), we get $a = 1, d = 2$

$\therefore S_n = \frac{n}{2}[2 + (n - 1)2] = n^2$

9. Let AB be the tower of height h m and AD be the flagstaff and C be the required point on the ground at the distance of x m from the tower.

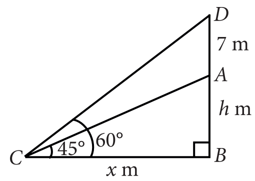
$\therefore AD = 7$ m

In $\Delta BCD, \tan 60° = \frac{BD}{BC} = \frac{h + 7}{x}$

$\Rightarrow \sqrt{3}x = h + 7 \Rightarrow x = \frac{h + 7}{\sqrt{3}}$ ...(i)

In ΔABC, $\tan 45° = \frac{AB}{BC} = \frac{h}{x}$

$\Rightarrow x = h$



$\Rightarrow \frac{h + 7}{\sqrt{3}} = h$ [Using (i)]

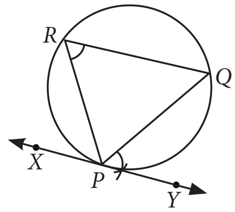
$\Rightarrow \sqrt{3}h = h + 7$

$\Rightarrow (\sqrt{3} - 1)h = 7$

$\Rightarrow h = \frac{7}{\sqrt{3} - 1} = \frac{7}{0.73} = 9.58 \approx 9.6$

Hence, height of tower is 9.6 m.

10. Given, radius of circle = 2.4 cm



Steps of construction

Step-I : Draw a circle of radius 2.4 cm and take a point P on the circle.

Step-II : Draw a chord PQ through the point P on the circle.

Step-III : Take a point R on the major arc and join PR and RQ.

Step-IV : On taking PQ as base, construct $\angle QPY = \angle PRQ$.

Step-V : Produce YP to X. Then, YPX is the required tangent at point P.

11. In the figure, let AB represent the light house.

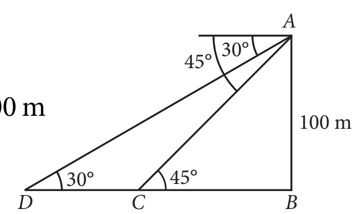
$\therefore AB = 100$ m

Let the positions of two ships be at C and D such that angle of depression from A are 45° and 30° respectively.

Now, in right ΔABC,

$\frac{AB}{BC} = \tan 45°$

$\Rightarrow \frac{100}{BC} = 1 \Rightarrow BC = 100$ m



Again, in right ΔABD, we have

$\frac{AB}{BD} = \tan 30°$

$\Rightarrow \frac{100}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 100\sqrt{3}$ m

The distance between the two ships = CD

$= BD - BC = 100\sqrt{3} - 100$

$= 100(\sqrt{3} - 1) = 100(1.732 - 1)$

$= 100 \times 0.732 = 73.2$ m

Thus, the required distance between the ships is 73.2 m.

12. Since it is given that mode = 36, which lies in the interval 30-40

$\therefore$ Modal class is 30-40.

$\therefore \ l = 30, f_1 = 16, \ f_0 = x, \ f_2 = 12, h = 10$

Now, $\text{Mode} = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$\Rightarrow \ 36 = 30 + \left(\dfrac{16 - x}{2 \times 16 - x - 12}\right) \times 10$

$\Rightarrow \ 36 = 30 + \left(\dfrac{16 - x}{20 - x}\right) \times 10$

$\Rightarrow (20 - x) \times 6 = (16 - x) \times 10$

$\Rightarrow 120 - 6x = 160 - 10x \Rightarrow 4x = 40 \Rightarrow x = 10$

OR

The frequency distribution table for the given data is as follows :

Class	Frequency (f_i)	Cumulative frequency (*c.f.*)
0-10	f_1	f_1
10-20	5	$f_1 + 5$
20-30	9	$f_1 + 14$
30-40	12	$f_1 + 26$
40-50	f_2	$f_1 + f_2 + 26$
50-60	3	$f_1 + f_2 + 29$
60-70	2	$f_1 + f_2 + 31$
Total	$31 + f_1 + f_2 = 40$	

Here, $N = 40 \Rightarrow 31 + f_1 + f_2 = 40$

$\Rightarrow f_1 + f_2 = 9$ ... (i)

Given, median = 32.5, which lies in the interval 30-40.

So, median class is 30-40.

$\therefore \ l = 30, h = 10, f = 12, N = 40$ and

$c.f. = f_1 + 14$

Now, $\text{median} = l + \left(\dfrac{\frac{N}{2} - c.f.}{f}\right) \times h$

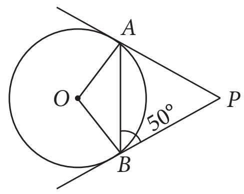
$\Rightarrow \ 32.5 = 30 + \left(\dfrac{20 - (f_1 + 14)}{12}\right) \times 10$

$\Rightarrow \ 2.5 = \left(\dfrac{6 - f_1}{12}\right) 10 \Rightarrow 6 - f_1 = \dfrac{2.5 \times 12}{10}$

$\Rightarrow \ 6 - f_1 = 3 \Rightarrow f_1 = 3$

From (i), $f_2 = 9 - 3 = 6$

13. (i)



Since, $OB \perp PB$ [Since, radius at the point of contact is perpendicular to tangent]

and $\angle PBA = 50°$ (Given)

$\therefore \ \angle OBA = 90° - 50° = 40°$

Also, $OA = OB$ [Radii of circle]

$\therefore \ \angle OAB = \angle OBA = 40°$

[Angle opposite to equal sides are equal]

(ii) Here, $\angle ABC = 90°$ (Angle in a semicircle)

So, in ΔABC, $\angle BAC = 180° - 90° - 55° = 35°$

Also, $\angle OAT = 90°$

$\Rightarrow \ \angle BAT + \angle OAB = 90° \Rightarrow \angle BAT = 90° - 35° = 55°$

14. (i) Quantity of ice-cream in the brick

= Volume of the brick = $(30 \times 25 \times 10)$ cm^3 = 7500 cm^3

$= \dfrac{7500}{1000} l$ [$\because$ 1 l = 1000 cm^3]

$= 7.5 \, l$

(ii) Volume of hemispherical scoop = $\dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3} \times \dfrac{22}{7} \times (3.5)^3 = \dfrac{1886.5}{21} = 89.83 \text{ cm}^3$

Now, volume of cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 3.5 \times 3.5 \times 15 = \dfrac{4042.5}{21} = 192.5 \text{ cm}^3$

$\therefore$ Number of scoops required to fill one cone

$= \dfrac{\text{Volume of a cone}}{\text{Volume of a scoop}} = \dfrac{192.5}{89.83} = 2.14 \approx 2$